M.Sc. Sem I Paper I

Abstract Algebra

THEOREM Prove that if a normal subgroup H of a group G is maximal then the quotient group G/H is simple.

Proof Let H is maximal.

we have to prove that the quotient group G/H is simple.

Definition of simple group : A group G having no proper normal subgroups is called a simple group. i.e. such simple group has only two normal subgroups namely G itself and the subgroup consisting of identity element 'e' only.

Let G/H is not simple

$\Rightarrow$ G/H has proper normal subgroups.

Let K/H be a proper normal subgroup of G/H.

[Use of this theorem]

If G be a group, H be a normal subgroup of G,
and K/H be a normal subgroup of G/H
then K is a normal subgroup of G
containing H.

$\Rightarrow$ Since K/H is a normal subgroup of G/H
So, K is a normal subgroup of G containing H.

But K/H is a proper subgroup of G/H
$\Rightarrow$ $H \subset K \subset G$.
Thus, K is a normal subgroup of G and
$H \subset K \subset G$.
$\Rightarrow$ By definition of maximal subgroups,
H is not maximal.
But we have supposed that H is maximal.
$\Rightarrow$ our supposition is wrong
$\Rightarrow$ G/H is simple.